College Prep Algebra
Lesson: Thursday, April 9, 2020

Learning Target:
Students will investigate e,using e in compounding continuously, and describe e

Let's Get Started:

Examine the picture on the next slide

What do you notice?

What do you wonder?

Practice: Complete the problems. Answers can be found on page 12.

A Dastardly Scheme

1) The formula for calculating compound interest is

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A	Amount of the Balance
P	Principle, the amount of money you start with
r	% rate **as a decimal.** $5\% = \frac{5}{100} = \mathbf{0.05}$
n	number of times compounded in a year
t	number of years money is in the account

Suppose you borrowed $1000 from a bank charging 12% annual interest. Complete the table to see how much money you would owe after 1 year based on the different options of compounding. (This table is based on NO PAYMENTS made to the bank at all for a year)

Use this link for a calculator. DESMOS Calculator

Number of times Compounded	Number of Times Interest is Calculated per Year	Expression for Balance on $1000 after 1 year	Account Balance on $1000 after 1 Year
Annual	1	$A = 1000\left(1+\frac{0.12}{1}\right)^{1\cdot 1}$	1120
Semi-Annual	2	$A = 1000\left(1+\frac{0.12}{2}\right)^{2\cdot 1}$	1123.60
Quarterly	4		
Monthly			

Use this link for a calculator. DESMOS Calculator

2) A dastardly accountant at the credit card company notices that if they charge interest more frequently, they will earn more money. (Sure, not much, but if they do it to everybody...) How frequently will they have to charge 12% interest in order for you to owe $1128 after one year?

Number of times Compounded	Number of Times Interest is Calculated per Year	Expression for Balance on $1000 after 1 year	Account Balance on $1000 after 1 Year
Weekly	52		
Daily	365		
Hourly			
Minute-ly			

What?! The accountant's tiny, cold heart skips a beat. There doesn't seem to be a way to even reach $1127.50! He thinks, what if we could charge interest *all the time*? Surely, if we could compound the interest infinite times per year, we could make lots of dollars. Sounds pretty amazing.

That was exactly what a banker was wondering.

Go to this YouTube video to learn about a banker that had the same idea. Stop once you get to time 5:30

Numberphile: e(Euler's Number)

The constant *e* is used in situations that involve ***continuous growth.***

Continuous compound growth is modeled with:

$$A = Pe^{rt}$$

...where P represents the initial value, r represents the growth rate, t represents units of time, and A represents the ending value.

Apply your knowledge:

1.) According to statistical surveys, the annual growth rate in the world population in recent years is about 1.7%. There were about 5.3 billion people living on this planet in 1990.

Unlike bank accounts, the compounding of population growth does not take place annually or quarterly. It's going on all the time. Every second of every hour, people are being born and others are dying. Thus, the growth rate (the overall effect of all of those births and deaths) can be viewed as a ***continuous process***.

If the same growth rate continues, how many people will inhabit the Earth by 2012?

Use $A = Pe^{rt}$, where $P = 5.3$, $r = 0.017$, and $t = 22$.

DESMOS Calculator

2.) Suppose you invest $1,050 at an annual interest rate of 5.5% compounded continuously. How much money, to the nearest dollar, will you have in the account after five years? (Financial institutions will not, in general, offer interest rates that are compounded continuously)

3.) A bacterial culture of 10,000 is growing continuously at a rate of 16% per day.

a.) Find the population of the culture at the end of one day.

b.) Find the population of the culture at the end of five days.

DESMOS Calculator

4.) If a population started with 20,000 members and grew continuously at a rate of 10% per year, how long would it take the population to double?

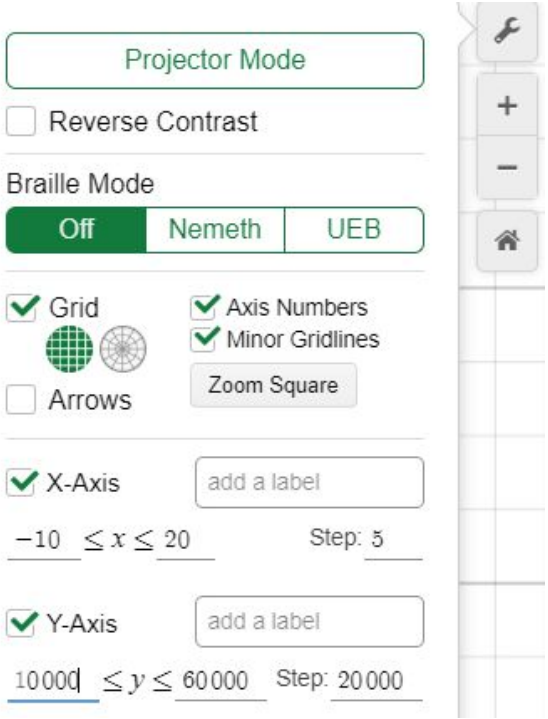
Type in $y = 20000e^{0.10x}$

Look for the *x* (time) where the *y* (population) has doubled in size on the graph. You can put your curser on the line and click to see the ordered pair of the graph.

Sketch the graph and your answer below.

DESMOS Calculator

When on DESMOS, click on the **Wrench** in the upper right corner of the screen and set your window like mine here.



5.) **Summary Time.** Now imagine that an interested and your 4th grade teacher approaches you and asks, "What is *e*?" What would you say to them?

Think before you write, and maybe even go back over the video. Describe it in your own words.

Answer Key:

Once you have completed the problems, check your answers here.

Answer Key
Exponentials and e

Additional Practice:

Click on the links below to get additional practice and to check your understanding!

Continuous Compound Interest